

The absence of QCD β -function factorization property of the generalized Crewther relation in the 't Hooft \overline{MS} -based scheme

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ABSTRACT

We apply the 't Hooft \overline{MS} -based scheme to study the scheme-dependence of the QCD generalization of Crewther relation for the product of the normalised non-singlet perturbative contributions to the e^+e^- -annihilation Adler function and to the Bjorken sum rule of the polarized lepton-nucleon deep-inelastic scattering process. We prove that after the transformations from the pure \overline{MS} -scheme to the 't Hooft scheme the characteristic \overline{MS} -scheme theoretical property of this relation, namely the factorization of the β -function in its conformal symmetry breaking part, disappears. Another “non-comfortable” theoretical consequence of the application of this prescription in $\mathcal{N} = 1$ SUSY QED model is mentioned. It is shown, that within the 't Hooft scheme the expansions of Green functions in terms of the Lambert function is simplified in high orders of perturbation theory. This may be considered as the attractive feature of the 't Hooft scheme, which manifest itself in high-order perturbative phenomenological applications.

PACS numbers: 11.10.Gh; 11.15-q; 11.25.Db; 11.30.Ly.

Keywords: Conformal symmetry breaking; Perturbation theory; Scheme dependence.

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1 Introduction

Among fundamental consequences of the conformal symmetry in the theory of strong interactions is the existence of the quark-parton model Crewther relation [1], namely

$$C_D^{NS} \times C_{Bjp} = 1 \quad . \quad (1)$$

Here C_D^{NS} is the normalised Green function, related to the characteristic of the e^+e^- -annihilation to hadrons process, i.e. non-singlet (NS) part of Adler D-function, defined as

$$D_A^{NS} = \left(3 \sum Q_f^2 \right) C_D^{NS} \quad (2)$$

where Q_f are the electric charges of quarks.

The term C_{Bjp} in Eq.(1) is the quark-parton expression for the Green function, which is proportional to the Bjorken sum rule for the deep-inelastic scattering of polarized leptons on nucleons, namely

$$S_{Bjp} = \int_0^1 \left[g_1^{ep}(x, Q^2) - g_1^{en}(x, Q^2) \right] dx = \frac{1}{6} \frac{g_A}{g_V} C_{Bjp} \quad (3)$$

Within perturbative QCD both Green functions under consideration can be written down as

$$C_D^{NS}(a_s) = 1 + d_0 a_s + d_0 \sum_{i=2}^N d_{i-1} a_s^i + O(a_s^{N+1}) \quad (4)$$

$$C_{Bjp}(a_s) = 1 + c_0 a_s + c_0 \sum_{i=2}^N c_{i-1} a_s^i + O(a_s^{N+1}) \quad (5)$$

where $a_s = \alpha_s/\pi$ obeys the following renormalization group equation

$$\mu^2 \frac{\partial a_s}{\partial \mu^2} \equiv \beta(a_s) = - \sum_{i \geq 0} \beta_i a_s^{i+2} \quad . \quad (6)$$

The application of renormalization procedure, which result in the appearance of the QCD β -function, leads to the violation of the conformal symmetry in QCD (for a recent review see [2]) and to the appearance in the r.h.s. of Eq.(1) of the additional conformal symmetry breaking (CSB) contribution Δ_{csb} :

$$C_D^{NS}(a_s(Q^2)) \times C_{Bjp}(a_s(Q^2)) = 1 + \sum_{i \geq 1} a_s^{i+1} \lambda_i = 1 + \Delta_{csb}(a_s(Q^2)) \quad (7)$$

Note that the absence of the a_s -term in Eq.(7) is the consequence of the validity of the variant of Eq.(1) in the conformal invariant limit. In this limit the related equation reads:

$$C_D^{NS}(a_s) \times C_{Bjp}(a_s)|_{c-i} = 1 \quad (8)$$

This limit is realised e.g. in the case of the QED approximation, when internal photon vacuum polarization diagrams are neglected [3]. It is called sometimes as perturbative quenched QED (pqQED) model. The existence of the expression of Eq.(8) leads to the cancellations between

the terms proportional to $d_0 Q_f^2 a$, $d_0 d_i Q_f^{2(i+1)} a^{i+1}$ in Eq.(4) and the similar terms proportional to $c_0 Q_f^2 a$, $c_0 c_i Q_f^{2(i+1)} a^{i+1}$ in Eq.(5) (where $a = \alpha/\pi$) in the product of these pqQED Green functions. In the case of QCD these contributions to c_k and d_k ($k \geq 0$), which are cancelling due to the property of the initial conformal symmetry, are labelled by $SU(N_c)$ factors C_F^{k+1} [4]. One of the consequences of Eq.(8) is that the approximations of Green functions, which enter into this equation, are scale- scheme-independent. This happens in view of the fact that in the conformal invariant limit the expansion parameter does not depend from any scale.

Among the consequences of Eq. (8) is the following identity:

$$d_0 = -c_0 \quad . \quad (9)$$

It leads to the absence of the a_s -term in Eq.(4).

The expression for Δ_{csb} -term in the generalized Crewther relation is known in the \overline{MS} -scheme [5], which is related to application of the dimensional regularization [6]. It can be written down as

$$\Delta_{csb}(a_s(Q^2)) = \left(\frac{\beta(a_s)}{a_s} \right) K^{\overline{MS}}(a_s); \quad (10)$$

where the factorized factor contains the renormalization-group β -function, defined in Eq.(6). The polynomial $K^{\overline{MS}}(a_s)$ has the following form

$$K^{\overline{MS}}(a_s) = a_s K_1^{\overline{MS}} + a_s^2 K_2^{\overline{MS}} + a_s^3 K_3^{\overline{MS}} + \dots \quad (11)$$

At the third order of perturbation theory the existence of this factorized form of Eq.(10) was discovered in Ref. [7], where the coefficients $K_1^{\overline{MS}}$ and $K_2^{\overline{MS}}$ were fixed analytically using the order $O(a_s^3)$ approximation for C_D^{NS} , evaluated in Ref. [8] and confirmed in Refs.[9, 10], and the similar perturbative approximation for $C_{Bjp}(a_s)$, obtained in Ref.[11]. Then the validity of the effect of β -function factorization in Eq.(10) was proved in all orders of perturbation theory using the \overline{MS} -scheme [12]. The concrete analytical expression of the coefficient $K_3^{\overline{MS}}$ was obtained in Ref.[13] using the evaluated $SU(N_c)$ group analytical expressions for the order a_s^4 coefficients in Eq.(4) and Eq.(5) in the \overline{MS} -scheme and the 3-loop \overline{MS} -scheme expression for $SU(N_c)$ -group β -function, evaluated in Ref. [14] and confirmed in Ref. [15].

It is necessary to remind that the coefficients of the β -function do not depend from the concrete realisation of the minimal subtraction scheme and are the same in the MS -scheme [16], \overline{MS} -scheme, or less known, but rather useful in practical calculations G -scheme [17]. However, the coefficients of the perturbation expansions of Green functions do depend from the concrete realisation of the minimal subtractions scheme. In our studies we will use the \overline{MS} -scheme results for Eq.(4), Eq.(5) and the related expression for the CSB-term of Eq.(10).

There is also another \overline{MS} representation for the CSB-term in Eq.(7), which is true at the $O(a_s^4)$ -level for sure and has the following form [18]

$$\Delta_{csb}(a_s) = \sum_{n \geq 1} \left(\frac{\beta(a_s)}{a_s} \right)^n \mathcal{P}_n(a_s) \equiv \sum_{n \geq 1} \sum_{r \geq 1} \left(\frac{\beta(a_s)}{a_s} \right)^n P_n^{(r)}[k, m] C_F^k C_A^m a_s^r \quad (12)$$

where $P_n^{(r)}[k, m]$ with $r + m = k$ do not depend from Casimir operators. Note, that at this 4-th order of perturbation theory the dependence from other $SU(N_c)$ group structure constants $d^{abcd} d^{abcd}$ is absent.

There are theoretical questions, which were not yet studied in the case of the representations of Eq.(10) and Eq.(12) for the generalized Crewther relation. One of them is whether there is some special theoretical information, which is encoded in Eq.(10) and Eq.(12) and whether the factorization feature is true in the \overline{MS} -scheme only. In this work for the analysis of this problem we apply the 't Hooft scheme [19], [20]. We clarify first how to use this scheme more rigorously in the phenomenologically oriented studies in high orders of perturbation theory. Next, we will reveal theoretical problems of the 't Hooft scheme, which are manifesting themselves in the studies of the generalized Crewther relation. Namely, we will demonstrate that the factorization property of Eq.(10) and Eq.(12) is not manifesting itself in the 't Hooft scheme and discuss possible theoretical explanation of this “uncomfortable” result, which may be compared with the troubles of its application in pure theoretical $\mathcal{N} = 1$ SUSY QED model.

2 The key points of the 't Hooft scheme definition

It is known that in gauge theories with single coupling constant the first two coefficients of the β -function, defined in Eq.(6), are scheme-independent. In the works of Refs. [19], [20] 't Hooft proposed to use in the theoretical studies the scheme, which is characterised by β -function with two scheme-independent coefficients only:

$$\mu^2 \frac{\partial a_H}{\partial \mu^2} = \beta(a_H) = -\beta_0 a_H^2 - \beta_1 a_H^3 \quad (13)$$

Indexes H are used for labelling 't Hooft-scheme parameters. The nullification of higher order coefficients of the β -function is achieved by finite renormalizations of charge. They are changing the expressions for the coefficients of perturbation theory series for Green functions, evaluated in the concrete renormalization scheme, say \overline{MS} -scheme.

This 't Hooft prescription depends on the choice of the initial scheme, which is used for the calculation of a Green function. In this work we consider massless perturbative series for two Green functions, defined by Eq.(4) and Eq.(5). The transformed to the 't Hooft scheme corresponding perturbative series can be written down as

$$C_D^{NS}(a_H) = 1 + d_0 a_H + d_0 \sum_{i=2}^N d_{i-1}^H a_H^i + O(a_H^{N+1}) \quad (14)$$

$$C_{Bjp}(a_H) = 1 + c_0 a_H + c_0 \sum_{i=2}^N c_{i-1}^H a_H^i + O(a_H^{N+1}) \quad (15)$$

Note that, like in the \overline{MS} -scheme, vector current is conserved in the 't Hooft scheme. Indeed, at the 2-loop level the renormalization of the vector current is the same as in the \overline{MS} -scheme, and therefore its divergency is zero. In higher orders of perturbation theory the conservation of the vector current in the 't Hooft scheme holds, since higher order corrections to the 't Hooft scheme β -function are identically equal to zero. Therefore, from this point of view the definition of this scheme is theoretically consistent.

Taking this into account one can safely write traditional inverse logarithmic expression for

the QCD coupling constant a_H . It reads

$$a_H = \frac{1}{\beta_0 L} - \frac{\beta_1 \ln L}{\beta_0^3 L^2} + \frac{\beta_1^2}{\beta_0^5 L^3} (\ln^2 L - \ln L - 1) - \frac{\beta_1^3}{\beta_0^7 L^4} (\ln^3 L + \frac{5}{2} \ln^2 L + 2 \ln L - \frac{1}{2}) + O(\frac{1}{L^5}) \quad (16)$$

where $L = \ln(Q^2/\Lambda_{\overline{MS}}^2)$. At this stage instead of the inverse log expansion of Eq.(16) it is possible to use the expression through the Lambert functions, which follows from the explicit solution of Eq.(13) and has the following form

$$a_H = -\frac{\beta_0^2}{\beta_1(1 + W_{-1}(z_w(L)))} \quad (17)$$

where $z_w(L) = (\beta_0^2/\beta_1)\exp(-1 + i\pi - (\beta_0^2/\beta_1)L)$ and W_k , $k = 0, \pm 1, \dots$, are the branches of the Lambert function, defined as the solution of the equation $z = W(z)\exp(W(z))$. This 2-loop expression is applied in the analysis of multiloop calculations starting from the works of Refs. [21, 22, 23, 24] and is used now rather regularly (see e.g. Refs.[25], [26], [27]). Moreover, following the ideas of the works of Refs.[25], [27], we propose to use Eq.(17) as the expansion parameter in the beyond-next-to-leading order QCD studies. However, this step should be done with care. Indeed, in this case it is necessary to take into account the re-calculated to the 't Hooft scheme coefficients d_i^H and c_i^H in the perturbative series of Eq.(14), Eq.(15) or of any other similar physical quantities. In the next section we clarify how to get them starting from the \overline{MS} -scheme results and present their explicit expressions up to 4-th order terms.

3 Determination of the perturbative expansions for Green functions in the 't Hooft scheme

Consider perturbation series for the Green function of Eq.(14). Within the effective-charges approach of Ref. [28] it can be re-written as

$$C_D^{NS} = 1 + d_0 a_{eff}^D$$

where the effective charge a_{eff}^D obeys the following renormalization group equation

$$\mu^2 \frac{\partial}{\partial \mu^2} a_{eff}^D = \beta(a_{eff}^D) = - \sum_{n \geq 0} \beta_n^{SI,D} a_{eff}^{D(n+2)} \quad (18)$$

with scheme-independent (SI), but Green-function dependent coefficients $\beta_n^{SI,D}$. They are related to scheme-invariants, introduced in Ref. [29]. Following the studies of Refs. [30], [31], we express them in the following form

$$\frac{\beta_n^{SI,D}}{n-1} = \frac{\beta_n}{n-1} + \beta_0 d_n - \beta_0 \frac{\Omega_n}{d_0} \quad (19)$$

where

$$\Omega_2 = d_0 d_1 \left(\frac{\beta_1}{\beta_0} + d_1 \right) \quad (20)$$

$$\Omega_3 = d_0 d_1 \left(\frac{\beta_2}{\beta_0} - \frac{1}{2} d_1 \frac{\beta_1}{\beta_0} - 2d_1^2 + 3d_2 \right). \quad (21)$$

$$\Omega_4 = \frac{d_0}{3} \left(3d_1 \frac{\beta_3}{\beta_0} + d_2 \frac{\beta_2}{\beta_0} - 4d_1^2 \frac{\beta_2}{\beta_0} + 2d_1 d_2 \frac{\beta_1}{\beta_0} - d_3 \frac{\beta_1}{\beta_0} + 14d_1^4 - 28d_1^2 d_2 + 5d_2^2 + 12d_1 d_3 \right). \quad (22)$$

It is possible to find the expressions for Ω_n (with $n \geq 4$) as well (Ω_n was obtained in Ref.[30]). At the present level of the development of calculations machinery it is enough to stop at this level. In fact taking into account Ω_4 -expression is already related to the not yet achieved in QCD 5-th perturbative level. However, its consideration will help to reveal some interesting features.

To get the expressions for the coefficients d_n in the 't Hooft scheme we use Eq.(19) and write down the following identity:

$$\frac{\beta_n^{\overline{MS}}}{n-1} + \beta_0 d_n^{\overline{MS}} - \beta_0 \frac{\Omega_n^{\overline{MS}}}{d_0} = \frac{\beta_n^H}{n-1} + \beta_0 d_n^H - \beta_0 \frac{\Omega_n^H}{d_0} \quad (23)$$

Taking into account that within 't Hooft scheme

$$\beta_n^H \equiv 0 \text{ at } n \geq 2 \quad (24)$$

and using Eq.(20)-Eq.(22), we find the following expressions for the coefficients of Green functions:

$$d_0 d_2^H = d_0 d_2^{\overline{MS}} + d_0 \frac{\beta_2^{\overline{MS}}}{\beta_0} \quad (25)$$

$$d_0 d_3^H = d_0 d_3^{\overline{MS}} + \frac{d_0}{2} \frac{\beta_3^{\overline{MS}}}{\beta_0} + 2d_0 d_1^{\overline{MS}} \frac{\beta_2^{\overline{MS}}}{\beta_0} \quad (26)$$

$$d_0 d_4^H = d_0 d_4^{\overline{MS}} + \frac{1}{3} d_0 \frac{\beta_4^{\overline{MS}}}{\beta_0} + d_0 d_1^{\overline{MS}} \frac{\beta_3^{\overline{MS}}}{\beta_0} + 3d_0 d_2^{\overline{MS}} \frac{\beta_2^{\overline{MS}}}{\beta_0} + \frac{5}{3} d_0 \left(\frac{\beta_2^{\overline{MS}}}{\beta_0} \right)^2 - \frac{1}{6} d_0 \frac{\beta_1 \beta_3^{\overline{MS}}}{\beta_0^2} \quad (27)$$

Absolutely identical formulae can be obtained for the coefficients c_i of Eq.(5) for the Bjorken polarized sum rule coefficient function C_{Bjp} . It is possible to use in the series of Eq.(14) and Eq.(15) the exact coupling constant expression through Lambert function from Eq.(17) after taking into account the evaluated correction terms. Therefore, on the first glance, the application of the 't Hooft scheme has attractive features of the simplification of the perturbative studies within several approaches, and in particular within the one proposed in Ref. [32] (for a detailed review see Ref.[33]).

4 The generalized Crewther relation and theoretical problems of the 't Hooft scheme

To find the structure of the generalized Crewther relation in the \overline{MS} -version of the 't Hooft scheme it is necessary to construct the product $C_D^{NS}(a_H)C_{Bjp}(a_H)$ of Eq.(14) and Eq.(15) with taking into account the explicit expressions of the obtained above coefficients $d_0 d_2^H$, $d_0 d_3^H$ (see Eq.(25) and Eq.(26)) and the similar expressions for the coefficients $c_0 c_2^H$, $c_0 c_3^H$. Using them together with the results for $d_0 d_1^{\overline{MS}}$ [34, 35, 36], $d_0 d_2^{\overline{MS}}$ [8, 9, 10], $d_0 d_3^{\overline{MS}}$ [13] and for the similar \overline{MS} -scheme terms in C_{Bjp} , obtained in Refs.[37, 38], Ref.[11] and Ref.[13], we come to the following statement:

Statement. In the 't Hooft \overline{MS} -based scheme there is no explicit factorization of the terms $(\beta(a_H)/a_H)$ in the QCD generalizations of Crewther relations of Eq.(10) and Eq.(12). This feature distinguishes it from \overline{MS} -scheme (or from any other version of MS -scheme) and

is raising the questions of applicability of the 't Hooft scheme for revealing theoretical effects, hidden in analytical high-order corrections to Green functions and β -function as well.

Proof. Let us compare the following representations for the CSB term in the generalized Crewther relation of Eq.(7)^c:

$$\Delta_{csb}^{\overline{MS}}(a_{\overline{MS}}) = (-\beta_0 a_{\overline{MS}} - \beta_1 a_{\overline{MS}}^2 - \beta_2^{\overline{MS}} a_{\overline{MS}}^3 - \beta_3^{\overline{MS}} a_{\overline{MS}}^4) \times \quad (28)$$

$$\times (a_{\overline{MS}} K_1^{\overline{MS}} + a_{\overline{MS}}^2 K_2^{\overline{MS}} + a_{\overline{MS}}^3 K_3^{\overline{MS}} + a_{\overline{MS}}^4 K_4^{\overline{MS}})$$

$$\Delta_{csb}^H(a_H) = (-\beta_0 a_H - \beta_1 a_H^2)(a_H K_1^H + a_H^2 K_2^H + a_H^3 K_3^H + a_H^4 K_4^H) \quad . \quad (29)$$

The analytical expressions for $K_1^{\overline{MS}}$, $K_2^{\overline{MS}}$ in Eq.(28) were obtained in Ref. [7], while the expression for $K_3^{\overline{MS}}$ was found in Ref.[13]. The possibility to add unknown term $K_4^{\overline{MS}}$ to the second part of Eq.(28) follows from the general proof of Ref. [12] of the validity of perturbative all-orders factorization of $(\beta(a_s)/a_s)$ factor in the CSB part of the \overline{MS} -scheme generalization of the Crewther relation.

Multiplying the series for $C_D^{NS}(a_H)$ and $C_{Bjp}(a_H)$ fom Eq.(14) and Eq.(15) and taking into account transformation formulae of Eq.(25), Eq.(26), and Eq.(27) for both $d_0 d_i^H$ and $c_0 c_i^H$ -terms, we find the expressions for K_1^H , K_2^H , K_3^H , K_4^H -terms. The results read

$$K_1^H = K_1^{\overline{MS}} \equiv K_1 \quad (30)$$

$$K_2^H = K_2^{\overline{MS}} \equiv K_2 \quad (31)$$

$$K_3^H = K_3^{\overline{MS}} + 3K_1 \frac{\beta_2^{\overline{MS}}}{\beta_0} \quad (32)$$

$$K_4^H = K_4^{\overline{MS}} + 4K_2 \frac{\beta_2^{\overline{MS}}}{\beta_0} + 2K_1 \frac{\beta_3^{\overline{MS}}}{\beta_0} \quad (33)$$

where the coefficients $\beta_2^{\overline{MS}}$ and $\beta_3^{\overline{MS}}$ of the QCD β -function are known from calculations of Refs.[14, 15] and Refs.[39, 40] correspondingly. Note that in QCD two latter equations cannot be simplified. There is no polynomials in Casimir operators in the terms K_3^H , K_4^H , etc. Further on, we can rewrite the expression for Eq.(29) in the following form

$$\Delta_{csb}^H(a_H) = \left(-\beta_0 a_H - \beta_1 a_H^2 \right) \left(K_1 a_H + K_2 a_H^2 + K_3^{\overline{MS}} a_H^3 + K_4^{\overline{MS}} a_H^4 \right) - \quad (34)$$

$$-3\beta_2^{\overline{MS}} K_1 a_H^4 - a_H^5 \left[\beta_0 K_4^{\overline{MS}} + 4K_2 \beta_2^{\overline{MS}} + 2K_1 \beta_3^{\overline{MS}} + 3K_1 \frac{\beta_1 \beta_2^{\overline{MS}}}{\beta_0} \right] + O(a_H^6)$$

One can see that there is no explicit factorization in 't Hooft renormalization procedure. It can be shown that the similar conclusion is true for another representation of the generalized Crewther relation of Eq.(12) as well.

5 Discussions

One may wonder what might be the reason for the drastic differences between the forms of the generalized Crewther relation in the \overline{MS} -scheme and the 't Hooft scheme. To our point of

^cFor the sake of generality, we included also non-calculated a_s^5 \overline{MS} -scheme corrections.

view the failure to reproduce factorized form of the \overline{MS} -scheme generalization of the Crewther relation is connected with the absence of diagrammatic representations of the corresponding results in the 't Hooft scheme. Indeed, while within \overline{MS} -scheme it is possible to understand the origin of the factorized form of Eq.(28) (see e.g. Ref.[12]), it seems impossible to formulate on the diagrammatic level the origin of the appearance of non-factorizable corrections to the generalized Crewther relation in the 't Hooft prescription. The similar problem arises if one tries to use it, say, in $\mathcal{N} = 1$ SUSY QED model. Indeed, it was shown in Ref.[41] that the application in this model of the based on the covariant derivative regularization approach [42], [43] SUSY invariant regularization [44], allows to understand on the diagrammatic language the origin of the existence of the scheme with β -function, expressed through anomalous dimension of superfield [45]. In the 't Hooft prescription of Eq.(13) this feature will be never seen. This observation, together with the fail to reproduce factorizable expression for the \overline{MS} -scheme variant of Crewther relation leads to the conclusion that one should not use 't Hooft prescription in the theoretical studies of the special features of gauge theories, which are manifesting themselves in the renormalization-group calculations, performed at the beyond-two-loop level. In principle, this was foreseen by 't Hooft himself in the work of Ref.[19], where he wrote “We do think perturbation theory up to two loops is essential to obtain accurate definition of the theory. But we were not able to obtain sufficient information on the theory to formulate self-consistent procedure for accurate computations”. This statement makes quite understandable the doubts on applicability of the 't Hooft scheme in asymptotic regime [46], related to applications of the perturbative results obtained beyond-two-loop approximation.

This forgotten statement of Ref.[19] is also supporting our non-comfortable feeling from the fail to reproduce factorizable expressions of the Crewther relation, explicitly obtained with the help of application of another 't Hooft prescription - the scheme of minimal subtractions for subtracting ultraviolet divergences [16] at the level of order a_s^4 corrections [13], [18].

Acknowledgements. One of us (AVG) is grateful to G. 't Hooft for his elucidations of historical aspects and arguments which led him to introducing the procedure called now by his name. ALK is grateful to D.V. Shirkov and D.I. Kazakov for encouraging to study the problem of scheme-dependence of Crewther relation and to M.J.C. Veltman for his help to get at hands the original work of Ref.[19] two decades ago. Both of us are grateful to K.V.Stepanyantz for interest to the work and useful comments. The work of one of us (AVG) comes from his Bachelor Thesis, done in Physics Department of Moscow State University. The work of ALK was supported in part by the RFBR grants No 11-01-00182, No 11-02-00112 and the grant NS-5525.2010.2.

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